BARI-TH/05-517

Model independent analysis of a class of $\overline{B}_s^0$ decay modes

P. Colangelo and R. Ferrandes

Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy
Dipartimento di Fisica, Universitá di Bari, Italy

Abstract

The widths of a class of two-body $\overline{B}_s^0$ decays induced by $b \to c\bar{u}d$ and $b \to c\bar{u}s$ transitions are determined in a model-independent way, using $SU(3)_F$ symmetry and existing information on $\bar{B} \to D_{(s)}P$ and $\bar{B} \to D_{(s)}V$ decays, with P and V a light pseudoscalar or vector meson. The results are relevant for the B_s physics programmes at the hadron colliders and at the e^+e^- factories running at the peak of $\Upsilon(5S)$.

In the next few years an intense B_s physics programme will be pursued at the hadron colliders, the Fermilab Tevatron and the CERN LHC, and at the e^+e^- factories running at $\Upsilon(5S)$. The programme includes precise determination of the $B_s - \overline{B}_s$ mixing parameters and search for CP violating asymmetries in B_s decays, with the aim of providing new tests of the Standard Model (SM) and searching for physics beyond SM. The analysis of rare B_s transitions is another aspect of the research programme, with the same aim of looking for deviations from SM expectations.

The knowledge of non leptonic B_s decay rates is of prime importance for working out the research programme. For example, $B_s - \overline{B}_s$ mixing can be studied using B_s two-body hadronic decay modes in addition to semileptonic modes. It is noticeable that the widths of a set of two-body transitions can be predicted in a model independent way, using the symmetries of QCD and available information on B decays. We are referring in particular to a class of decay modes induced by the quark transitions $b \to c\bar{u}d$ and $b \to c\bar{u}s$, for example those collected in Table 1. The key observation is that the various

Table 1: $SU(3)$ decay amplitudes for $\overline{B}^0_s \to D_{(s)}P$ decays, with P a light pseudoscalar meson. In the last column the corresponding branching fractions predicted using the method described in the text are reported.

decay mode	amplitude	BR
$\overline{B}^0_s \to D^+_s \pi^-$	$V^*_{ud}V_{cb}\ T$	$(2.9 \pm 0.6) \times 10^{-3}$
$\overline{B}^0_s \to D^0 \overline{K}^0$	$V^*_{ud}V_{cb}\ C$	$(8.1 \pm 1.8) \times 10^{-4}$
$\overline{B}^0_s \to D^0 \eta_8$	$\frac{1}{\sqrt{6}}\ V^*_{us}V_{cb}\ (2C - E)$	
$\overline{B}^0_s \to D^0 \eta_0$	$V^*_{us}V_{cb}\ D$	
$\overline{B}^0_s \to D^0 \eta$		$(2.1 \pm 1.2) \times 10^{-5}$
$\overline{B}^0_s \to D^0 \eta'$		$(9.8 \pm 7.6) \times 10^{-6}$
$\overline{B}^0_s \to D^0 \pi^0$	$-\frac{1}{\sqrt{2}}\ V^*_{us}V_{cb}\ E$	$(1.0 \pm 0.3) \times 10^{-6}$
$\overline{B}^0_s \to D^+ \pi^-$	$V^*_{us}V_{cb}\ E$	$(2.0 \pm 0.6) \times 10^{-6}$
$\overline{B}^0_s \to D^+_s K^-$	$V^*_{us}V_{cb}\ (T + E)$	$(1.8 \pm 0.3) \times 10^{-4}$

decay modes are governed, in the $SU(3)_F$ limit, by few independent amplitudes that can be constrained, both in moduli and in phase differences, from corresponding B decay processes.

Considering transitions with a light pseudoscalar meson belonging to the octet in the final state, the scheme where the correspondence can be established involves the three

different topologies in $\overline{B}^0_s$ decays induced by $b \to c\bar{u}d(s)$, namely the color allowed topology T, the color suppressed topology C and the W-exchange topology E. The transition in the $SU(3)$ singlet η_0 involves another amplitude D in principle not related to the previous ones. Notice that the identification of the different amplitudes is not graphical, it is based on $SU(3)$ [1]. Since $\overline{B} \to DP$ decays induced by the quark processes $b \to c\overline{u}q$ ($q = d$ or s) involve a weak Hamiltonian transforming as a flavor octet, using de Swart's notation $T^{(\mu)}_{\nu}$ for the $\nu = (Y, I, I_3)$ component of an irreducible tensor operator of rank (μ) [2], one can write: $H_W = V_{cb}V^*_{ud}T^{(8)}_{0\,1\,-1} + V_{cb}V^*_{us}T^{(8)}_{-1\,\frac{1}{2}\,-\frac{1}{2}}$. When combined with the initial $\overline{B}$ mesons, which form a (3^*)-representation of SU(3), this leads to (3^*), (6) and (15^*) representations. These are also the representations formed by the combination of the final octet light pseudoscalar meson and triplet D meson. Therefore, using the Wigner-Eckart theorem for SU(3), the decay amplitudes can be written as linear combinations of three reduced amplitudes $\langle\phi^{(\mu)}|T^{(8)}|B^{(3^*)}\rangle$, with $\mu = 3^*, 6, 15^*$, which are independent of the quantum numbers Y, I, I_3 of the Hamiltonian and the initial and final states. By appropriate linear combinations of the three reduced amplitudes one can obtain a correspondence with the three topological diagrams of the various decay modes. The combinations correspond to C, T and E in Table 1, i.e. the color suppressed, color enhanced and W-exchange diagrams, respectively. The $SU(3)$ representation for B decays is reported in Table 2.

Table 2: SU(3) parameterization of $\bar{B} \to D_{(s)}P$ decay amplitudes induced by the $b \to c\bar{u}d$ and $b \to c\bar{u}s$ transitions, together with the experimental results reported by the Particle Data Group [3].

decay mode	amplitude	BR [3]
$B^- \to D^0\pi^-$	$V^*_{ud}V_{cb}\ (C+T)$	$(4.98 \pm 0.29) \times 10^{-3}$
$\overline{B}^0 \to D^0\pi^0$	$\frac{1}{\sqrt{2}}V^*_{ud}V_{cb}\ (C-E)$	$(2.91 \pm 0.28) \times 10^{-4}$
$\overline{B}^0 \to D^+\pi^-$	$V^*_{ud}V_{cb}\ (T+E)$	$(2.76 \pm 0.25) \times 10^{-3}$
$\overline{B}^0 \to D^+_sK^-$	$V^*_{ud}V_{cb}\ E$	$(3.8 \pm 1.3) \times 10^{-5}$
$\overline{B}^0 \to D^0\eta_8$	$-\frac{1}{\sqrt{6}}V^*_{ud}V_{cb}\ (C+E)$	
$\overline{B}^0 \to D^0\eta_0$	$V^*_{ud}V_{cb}\ D$	
$\overline{B}^0 \to D^0\eta$		$(2.2 \pm 0.5) \times 10^{-4}$
$\overline{B}^0 \to D^0\eta'$		$(1.7 \pm 0.4) \times 10^{-4}$
$B^- \to D^0K^-$	$V^*_{us}V_{cb}\ (C+T)$	$(3.7 \pm 0.6) \times 10^{-4}$
$\overline{B}^0 \to D^0\overline{K}^0$	$V^*_{us}V_{cb}\ C$	$(5.0 \pm 1.4) \times 10^{-5}$
$\overline{B}^0 \to D^+K^-$	$V^*_{us}V_{cb}\ T$	$(2.0 \pm 0.6) \times 10^{-4}$

Considering Table 2 one realizes that the three $\bar{B} \to DK$ experimental rates could allow to obtain $|T|$, $|C|$ and the phase difference $\delta_C - \delta_T$. This was already observed in [4], and can be recast in the determination of the two independent isospin amplitudes A_1 and A_0 for $I = 1$ and $I = 0$ isospin DK final states: $\mathcal{A}(B^- \to D^0 K^-) = \sqrt{2} A_1$, $\mathcal{A}(\bar{B}^0 \to D^+ K^-) = \frac{1}{\sqrt{2}}(A_1 + A_0)$ $\mathcal{A}(\bar{B}^0 \to D^0 \bar{K}^0) = \frac{1}{\sqrt{2}}(A_1 - A_0)$. Taking into account

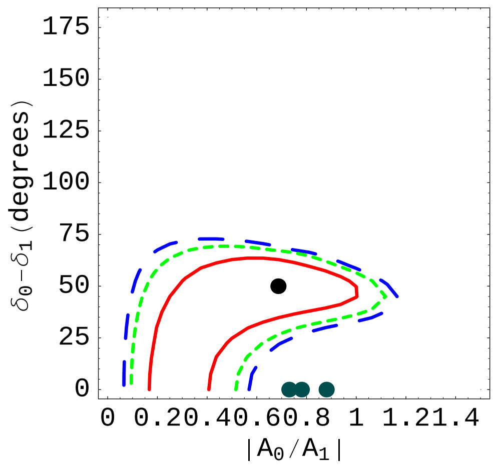

Figure 1: Ratios of isospin amplitudes A_0/A_1 for $\bar{B} \to DK$ transitions obtained from data in Table 2. The contours correspond to the confidence level of 68% (continuous line), 90% (dashed line) and 95% (long-dashed line), with the dots inside showing the result of the fit. The dots along the x axis correspond to the results of naive factorization.

the difference of the B^- and $\bar{B}^0$ lifetimes: $\tau_{B^-} = 1.671 \pm 0.018$ ps and $\tau_{B^0} = 1.537 \pm 0.014$ ps, but neglecting the tiny phase space correction due to the difference between $p_{D^0 K^-} = p_{D^0 \bar{K}^0} = 2280$ MeV and $p_{D^+ \bar{K}^-} = 2279$ MeV, with p the modulus of the three-momentum of one of the two final mesons in the B rest frame, one would obtain allowed region for A_0/A_1 at various confidence levels by minimizing the χ^2 function for the three branching ratios and plotting the χ^2 contours that correspond to a given confidence level, as done in fig.1. Due to the quality of the experimental data and to the correlation between $|A_0/A_1|$ and $\delta_0 - \delta_1$, the allowed region is not tightly constrained, in particular the phase difference could be zero.

We pause here, since we can elaborate once more about factorization approximations sometimes adopted for computing non leptonic decays, in this case for B mesons [5]. In fig.1 we have shown the predictions by, e.g., naive factorization, where the decay amplitudes are written in terms of K and D meson leptonic constants f_K and f_D, and

the $B \to D$ and $B \to K$ form factors F_0: $\mathcal{A}(\bar{B}^0 \to D^+K^-)_F = i\frac{G_F}{\sqrt{2}}V_{cb}V^*_{us}a_1(m_B^2 - m_D^2)f_K F_0^{B\to D}(m_K^2)$ and $\mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0)_F = i\frac{G_F}{\sqrt{2}}V_{cb}V^*_{us}a_2(m_B^2 - m_K^2)f_D F_0^{B\to K}(m_D^2)$. The result of this approach corresponds to vanishing phase difference; using $a_1 = c_1 + c_2/3$ and $a_2 = c_2 + c_1/3$, with c_1 and c_2 the Wilson coefficients appearing in the effective hamiltonian inducing the decays (for their numerical values we quote $a_1 = (1.036, 1.017, 1.025)$ and $a_2 = (0.073, 0.175, 0.140)$ at LO and at NLO (in NDR and HV renormalization schemes) accuracy, respectively [6]) we obtain results corresponding to the dots along the horizontal axis in fig. 1, which do not belong to the region permitted by experimental data at 95% CL. In generalized factorization, where a_1 and a_2 are considered as parameters, the phase difference is constrained to be zero, too. This is allowed by the experimental data on these three channels, but excluded if one considers all channels, as we shall see below.

Coming to bounding the decay amplitudes, the four $\bar{B} \to D\pi$ and $\bar{B} \to D_s K$ decay rates cannot determine C, T, E and their phase differences [7]. $\bar{B} \to D_s K$ only fixes the modulus of E, which is not small at odds with the expectations by factorization, where W-exchange processes are suppressed by ratios of decay constants and form factors and are usually considered to be negligible. Moreover, the presence of E does not allow to directly relate color favoured T or color suppressed C decay amplitudes in $D\pi$ and DK final states. What can be done, however, is to use all the information on $\bar{B} \to D\pi, D_s K$ and DK (7 experimental data) to determine T, C and E (5 parameters). A similar attitude has been recently adopted in [8]. Noticeably, the combined experimental information is enough accurate to tightly determine the ranges of variation for all these quantities. In fig. 2 we have depicted the allowed regions in the C/T and E/T planes, obtained fixing the other variables to their fitted values, with the corresponding confidence levels. It is worth noticing that the phase differences between the various amplitudes are close to be maximal; this signals again deviation from naive (or generalized) factorization, provides contraints to QCD-based approaches proposed to evaluate non leptonic B decay amplitudes [9, 10, 11] and points towards sizeable long-distance effects in C and E [12, 13]. To obtain the amplitudes we have fixed the ratio $|V_{us}/V_{ud}|$ to the experimental result: $|V_{us}/V_{ud}| = 0.226 \pm 0.003$, and we have taken into account the phase space correction due to p_{DK}, $p_{D\pi} = 2306$ MeV and $p_{D_sK^-} = 2242$ MeV. We obtain $|\frac{C}{T}| = 0.53 \pm 0.10$, $|\frac{E}{T}| = 0.115 \pm 0.020$, $\delta_C - \delta_T = (76 \pm 12)^\circ$ and $\delta_E - \delta_T = (112 \pm 46)^\circ$. We have to mention that the accuracy of the fit is not particularly high since $\chi^2/dof = 2.3$, i.e. a fit probability of 10%. This is entirely due to a single entry in Table 2, the branching fraction of $B^- \to D^0K^-$. We have reported the PDG value corresponding to the average of two measurements, by CLEO ($(2.92 \pm 0.84 \pm 0.28) \times 10^{-4}$) and Belle ($(4.19 \pm 0.57 \pm 0.40) \times 10^{-4}$)

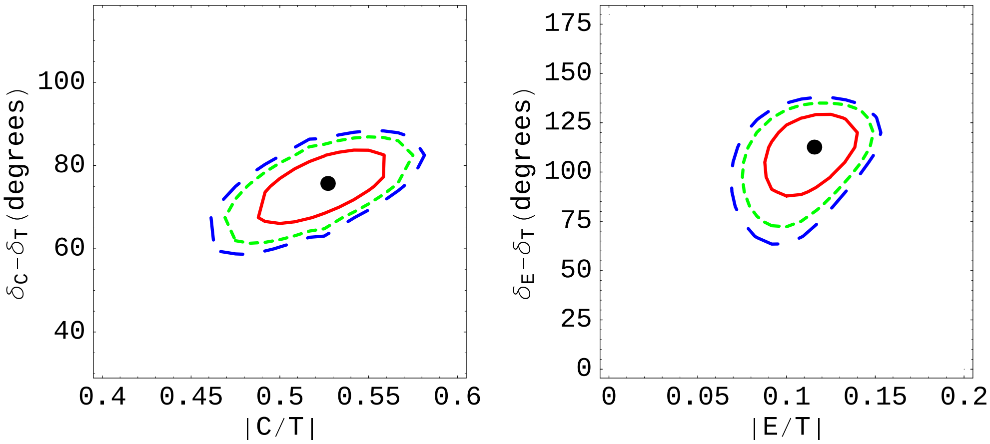

Figure 2: Ratios of SU(3) amplitudes obtained from data in Table 2. The contours correspond to the confidence level of 68% (continuous line), 90% (dashed line) and 95% (long-dashed line); the dots show the result of the fit.

Collaborations, with the error including a scale factor 1.1. The fit favours a smaller value: for example, using the CLEO result the χ^2/dof drops to 0.3 without sensibly modifying the results.

With the results for the amplitudes we can determine a number of B_s decay rates, and the predictions are collected in Table 1. The uncertainties in the predicted rates are small; in particular, the W-exchange induced processes $\overline{B}^0_s \to D^+\pi^-, D^0\pi^0$ are precisely estimated [14].

Considering the decays with η or η' in the final state, they involve the amplitude D corresponding to the transition in a $SU(3)$ singlet η_0, and the $\eta-\eta'$ mixing angle θ (in a one angle mixing scheme):

$$
\begin{aligned}
A(\overline{B}^0 \to D^0\eta) &= \cos\theta[-\frac{1}{\sqrt{6}}(C+E)] - \sin\theta\frac{D}{\sqrt{3}} \\
A(\overline{B}^0 \to D^0\eta') &= \sin\theta[-\frac{1}{\sqrt{6}}(C+E)] + \cos\theta\frac{D}{\sqrt{3}} \quad . \qquad (1)
\end{aligned}
$$

If we use the value $\theta = -15.4^0$ for the mixing angle [15], we obtain $|\frac{D}{T}| = 0.41 \pm 0.11$ without sensibly constraining the $D-T$ phase difference, $\delta_D - \delta_T = -(25 \pm 51)^\circ$. Corresponding $\overline{B}^0_s$ decay rates are predicted consequently.

The key of the success of the programme of predicting B_s decay rates is the small number of amplitudes in comparison to the available data, a feature which is not common

to all processes. Considering $b \to c\bar{u}d(s)$ induced transitions, one could look at the case of one light vector meson in the final state, with the same $SU(3)$ decomposition reported in Tables 1, 2 (we denote by a prime the amplitudes involved in this case). B decay data are collected in Table 3. The difference with respect to the previous case is that the W-exchange mode $\bar{B}^0 \to D_s^+ K^{*-}$ has not been observed, yet, therefore the E' amplitude is poorly determined considering only the other modes. Taking into account phase space corrections due to $p_{D\rho} = 2235$ MeV and $p_{DK^*} = 2211$ MeV, we obtain $|\frac{C'}{T'}| = 0.36 \pm 0.10$, $|\frac{E'}{T'}| = 0.29 \pm 0.37$, $\delta_{C'} - \delta_{T'} = (48 \pm 67)^\circ$ and $\delta_{E'} - \delta_{T'} = (96 \pm 56)^\circ$, with a fit probability of 85%. The allowed region in the C'/T' plane, fixing all the other variables to their fitted values, is depicted in fig.3. In this case the phase difference $\delta_{C'} - \delta_{T'}$ can vanish. The

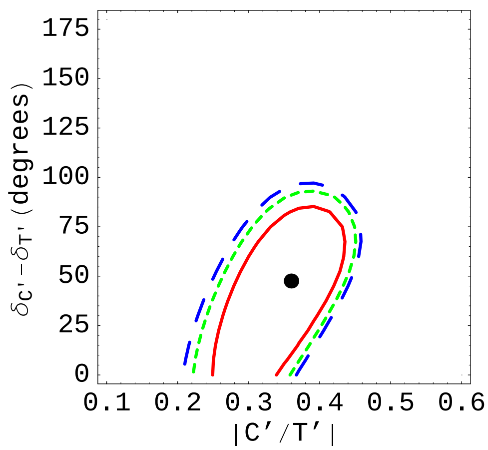

Figure 3: Ratio of C' and T' amplitudes for $\bar{B} \to DV$ transitions. The contours correspond to the same confidence level as in fig. 2; the dot shows the result of the fit.

predictions for $\overline{B}_s^0$ decay rates are collected in Table 3: as anticipated, the accuracy is not high for $W-$exchange induced decays. On the other hand, the prediction for the rate of $\overline{B}^0 \to D_s K^{*-}$: $\mathcal{B}(\overline{B}^0 \to D_s K^{*-}) = (1 \pm 5) \times 10^{-4}$, is compatible with the upper bound in Table 3.

Considering other decay modes induced by the same quark transitions, namely $\bar{B} \to D^*_{(s)} P$ and $\bar{B} \to D^*_{(s)} V$ decays, the present experimental data are not precise enough to sensibly constrain the independent amplitudes and to provide stringent predictions for B_s. As soon as the experimental accuracy will improve, a similar analysis will be possible to describe $\overline{B}_s^0 \to D^*_{(s)} P$ modes, while the three helicity $\bar{B} \to D^*_{(s)} V$ amplitudes will be needed to determine the corresponding B_s decays.

Table 3: Experimental results for the branching fractions of $\bar{B} \to D_{(s)}V$ decays induced by the $b \to c\bar{u}d$ and $b \to c\bar{u}s$ transitions as reported by the Particle Data Group [3]. The predictions for $\overline{B}^0_s$ decays obtained using the method described in the text are reported in the last column.

decay mode	BR [3]	decay mode	BR
$B^- \to D^0\rho^-$	$(1.34 \pm 0.18) \times 10^{-2}$	$\overline{B}^0_s \to D_s^+\rho^-$	$(7.2 \pm 3.5) \times 10^{-3}$
$\overline{B}^0 \to D^0\rho^0$	$(2.9 \pm 1.1) \times 10^{-4}$	$\overline{B}^0_s \to D^0\overline{K}^{*0}$	$(9.6 \pm 2.4) \times 10^{-4}$
$\overline{B}^0 \to D^+\rho^-$	$(7.7 \pm 1.3) \times 10^{-3}$		
$\overline{B}^0 \to D_sK^{*-}$	$< 9.9 \times 10^{-4}$		
$B^- \to D^0K^{*-}$	$(6.1 \pm 2.3) \times 10^{-4}$	$\overline{B}^0_s \to D^0\rho^0$	$(0.28 \pm 1.4) \times 10^{-4}$
$\overline{B}^0 \to D^0\bar{K}^{*0}$	$(4.8 \pm 1.2) \times 10^{-5}$	$\overline{B}^0_s \to D^+\rho^-$	$(0.57 \pm 2.8) \times 10^{-4}$
$\overline{B}^0 \to D^+K^{*-}$	$(3.7 \pm 1.8) \times 10^{-4}$	$\overline{B}^0_s \to D_s^+K^{*-}$	$(4.5 \pm 3.1) \times 10^{-4}$

Let us finally comment on the possible role of $SU(3)_F$ breaking terms that can modify our predictions. Those effects are not universal, and in general cannot be reduced to well defined and predictable patterns without new assumptions. Their parametrization would introduce additional quantities [16] that at present cannot be sensibly bounded since their effects seem to be smaller than the experimental uncertainties. Therefore they can be neglected until the experimental errors remain at the present level. It will be interesting to investigate their role when the B_s decay rates will be measured and more precise B branching fractions will be available.

Acnowledgments

We thank F. De Fazio for discussions. We acknowledge partial support from the EC Contract No. HPRN-CT-2002-00311 (EURIDICE).

References

[1] D. Zeppenfeld, Z. Phys. C **8** (1981) 77; B. Grinstein and R. F. Lebed, Phys. Rev. D **53** (1996) 6344.

[2] J. J. de Swart, Rev. Mod. Phys. **35** (1963) 916.

[3] S. Eidelman *et al.* [Particle Data Group], Phys. Lett. B **592** (2004) 1.

[4] Z. z. Xing, Eur. Phys. J. C **28** (2003) 63; C. W. Chiang and J. L. Rosner, Phys. Rev. D **67** (2003) 074013.

[5] For discussions about naive and generalized factorization see M. Neubert and B. Stech, Adv. Ser. Direct. High Energy Phys. **15** (1998) 294.

[6] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125.

[7] The isospin analysis and the discussion of the strong phases for $B \to D\pi$ transitions can be found in M. Neubert and A. A. Petrov, Phys. Lett. B **519** (2001) 50; Z. Z. Xing, High Energy Phys. Nucl. Phys. **26** (2002) 100; H. Y. Cheng, Phys. Rev. D **65** (2002) 094012; L. Wolfenstein, Phys. Rev. D **69** (2004) 016006.

[8] C. S. Kim, Sechul Oh, Chaehyun Yu, arXiv:hep-ph/0412418.

[9] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591** (2000) 313.

[10] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. **87** (2001) 201806.

[11] Y. Y. Keum, T. Kurimoto, H. N. Li, C. D. Lu and A. I. Sanda, Phys. Rev. D **69** (2004) 094018.

[12] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B **542** (2002) 71; Phys. Rev. D **69** (2004) 054023; Phys. Lett. B **597** (2004) 291; H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D **71** (2005) 014030.

[13] C. K. Chua and W. S. Hou, arXiv:hep-ph/0504084.

[14] For estimates of color enhanced decay rates based on the factorization ansatz see R. Aleksan, I. Dunietz and B. Kayser, Z. Phys. C **54** (1992) 653.

[15] T. Feldmann, P. Kroll, B. Stech, Phys. Rev. D **58** (1998) 114006.

[16] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **52** (1995) 6356.